MEASUREMENT TIME AND SPATIAL VARIABILITY OF FIELD INFILTRATION

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A b s t r a c t. Infiltration studies were performed on arenic chemozems of quarternality fluvial terrace of Labe (Elbe). We studied the applicability of infiltration equations of Philip (2-parametric), and 3-parametric, of Brutsaert and Swartzendruber, using 70 infiltration tests performed on a regular grid on a plot covered for 4 years by grass. The best applicability was found for the 3-parameter equations, but there is no unique recommendation. The probability density function (PDF) of estimates of soil hydraulic characteristics is deformed by errors of estimates due to the approximate character of equations used. The log-normal distribution is a well-acceptable approximation for sorptivity S, saturated hydraulic conductivity Ks and rates of infiltration. Only for some estimates of hydraulic functions a weak spatial variability was found for the distance of 7.5 m. Long time variability of infiltration and its parameters after ploughing and subsequent grass planting was studied on the other plot. There was a significant drop of Ks just after one season of grass growth and harvests. Low soil structural stability was indicated by the infiltration tests.

K e y w o r d s: field infiltration, arenic chemozems, infiltration equations

INTRODUCTION

All transport phenomena, the flow of water in soil and the accumulation, transformation and transport of the actual and potential pollutants, as well as gas diffusion, are related to the soil porous media for given boundary conditions. The quality of these media is dependent upon the structural development of soils. The primary and secondary aggregation is, e.g., reflected by the soil water retention curve and by the unsaturated conductivity function [7]; the nature of the structure is links to the type of the model of the soil porous system [6]. For the estimation of some soil hydraulic functions the inverse solution of infiltration is applicable as an expedient and fast method. Hydraulic functions and parameters of infiltration are then used for the quantitative discussion on the dynamics of soil structure. Our studies were, therefore, aimed at the evaluation of the field infiltration tests. The main problems were:

 Field testing of quasi-analytical and approximative equations of infiltration.

2. Application of the tested equations to the study on the long time alteration of the soil fabric and porous system after ploughing.

The space variability is restricted to a pedologically homogeneous district. This heterogeneity can be described as stochastic within the frame of the deterministic homogeneity of the pedotop.

METHODS

Site description and methods of measurement

The experimental station is situated at Tišice, district Mělnik, in Central Bohemia (about 20 km north-west from Prague). The total area of about 0.5 ha is divided into 8 plots, each of 52×68 m. The experimental plots lie on a quaternaly fluvial terrace of gravel sand, overlayed by permeable sand. The landscape is practically a plane, with slight irregular undulations.

Climatologically, the area belongs to the

warm region, dry with moderate winter. The total mean annual precipitation is 518.8 mm, precipitation during vegetation season (from April to November) - 343 mm. The mean annual temperature is $8.6 \,^{\circ}$ C (vegetation season 14.8 $^{\circ}$ C).

The whole experimental area is covered by arenic chernozems of the carbonate variety (FAO). The thickness of horizons and subhorizons is variable due to the fluvial influence upon the otherwise homogeneous sands of the parent material of the C horizon.

The sandy-loamy topsoil with about 2.5 % of humus and 15 % of particles under 0.002 mm reaches down to the depth of 40-80 cm. Below it is loamy sand with some loamy lenses (at about 60-100 cm) and permeable coarse sands (below 100 cm). The ground water table is about 4-5 m below the surface.

Two experimental plots have been chosen: Plot A was covered by grass (*Dactylis glomerata*), where grass had been grown during preceding 4 years. On plot B, the time variability of infiltration characteristics was studied. The first set of infiltration tests was performed just after ploughing and basic agricultural cultivation before sowing of grass in April. The second set of infiltrations was realized in autumn, after 5 months of growth of grass and harvesting. The third set of infiltrations was in spring the following year.

The spatial arrangement of tests in plots A and B differed slightly, mainly in the number of tests.

On plot A, a series of 70 infiltration tests was performed at equidistant positions. The distance between the centers of the infiltration rings was 7.5 m, the rectangular mesh was formed by 7 columns and 10 rows. Duration of the tests was 3.600 s each to avoid the irregularities earlier observed in long time infiltration. They were caused mainly by the divergence of flowpaths on the transition between Aph horizon with the root system and the subhorizon Ah(c) without distinct root system and with the increase of clay particles at some instances. The diameter of the inner ring was 37.5 cm, and that of the outer ring - 60 cm. The positive head of 2 cm ± 0.5 cm was held in both rings. Infiltration was measured in September at initial soil water content $\theta_i=0.222$ up to 0.236, at the depth 15-20 cm, and it was practically constant during the time period of measuring, with slight space variation only. In addition to infiltration tests, saturated hydraulic conductivity K_s was estimated by two further methods. K_s was then evaluated by the following methods:

- a) Ponded infiltration by Brutsaert's [1] equation (see later).
- b) Laboratory estimates by falling head permeameter with hydraulic gradient << 1, volume of core samples was 100 cm³, samples were taken from a layer of 5-10 cm below surface.
- c) Measurements with Guelph permeameter in shallow boreholes of 15 cm in depth and 8 cm in diameter. The water level in the borehole was kept at 10 cm above the bottom. The evaluation followed the method of Reynolds and Elrick [11].

On plot B, repeated series of 20 infiltration tests were performed. A rectangular net with a grid of 14 m was used in order to exclude totally the spatial dependence and thus the deformation of variance. At the first series in the spring, the surface was without wegetation, the soil was very loose after agrotechnical cultivation. The second series was done in autumn on the plot already covered by grass, the third series was similar, but in the spring the following year. Vegetation cover was regularly harvested and it was without irrigation. The duration of infiltration tests was chosen to be 1.800 s due to the sharp boundary between the cultivated and non cultivated part of Ah horizon. There, the flow paths of water diverged extremely when the wetting front reached the boundary. The time of 1.800 s was short enough to prevent this effect. Otherwise, infiltration tests were performed in the same way as on plot A.

Evaluation of infiltration tests

During the infiltration tests, the cumulative infiltration I (cm) was read in time t (min). Experimental data were fitted to the following infiltration equations which belong to the subclass

of approximative equations with physical interpretation of parameters.

Philip's [9] algebraic two-parameter equation is:

$$I = S_p t^{\nu_2} + At \tag{1}$$

the infiltration rate v = dI/dt is:

$$v = \frac{1}{2} S_p t^{\nu_2} + A \tag{2}$$

 S_p (L·T^{-1/2}) is the approximation of sorptivity S which is the first term in time series solution of Philip [8]. Owing to the truncation of infinite series solution, the truncation error ε_p can be neglected and S_p is then considered as a good estimate of sorptivity [5]. Parameter A (LT⁻¹) is $(A_2 + K_i + \varepsilon_A)$ where A_2 is the second term in the time series solution, K_i is the unsaturated conductivity at initial water content θ_i , ε_A is the truncation error, $\varepsilon_A > \varepsilon_p$. It is supposed that $A = m K_{1}$, where the value of m is in the range between 0.2 and 0.67 [10]. Detailed studies show that *m* depends upon θ , and t, and sometimes exceeds a theoretical upper limit of 2/3. The error of estimate of K. derived from A could theoretically reach about 30 % in homogeneous soil with $d\theta/dt = 0$ [5].

Three parameters equation of Kutilek and Krejča [4] was derived from the time series solution of Philip [8]:

$$I = C_1 t^{\nu_2} + C_2 t + C_3 t^{\nu_2} \tag{3}$$

where C_1 is the estimate of sorptivity, then $C_1=S_K$ and $S=S_K+\varepsilon_K$ where $\varepsilon_K < \varepsilon_p$, C_2 is the estimate of $(A_2+K_1+\varepsilon_2)$ and C_3 is the value of $(A_3+\varepsilon_3)$. Here, A_3 is the third term in the time series solution of Philip. If we approximate the limiting time for which the truncated Eq. (3) converges as the value of t_{lim} when $dv/dt \rightarrow 0$, we have :

$$t_{\rm lim} = \frac{C_1}{3C_3} \tag{4}$$

and with the additional approximation $v(t_{\lim}) \approx K_s$ we obtain the estimate:

$$K_s \approx (3 C_1 C_3)^{1/2} + C_2$$
 (5)

The extension of (1) to the three-parameter Eq. (3) reduced substantially the truncation errors in the first two terms; the estimation of S and K, is theoretically very good with errors below 10 %. The same is true for the next two 3-parameters equations.

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In order to validate Philip's time series equation to $t \rightarrow \infty$, Swartzendruber [12] proposed the exponential form of I(t) and, truncating all terms in exponential time series except the first one, he proposed:

$$I = \frac{S_s}{A_o} \left[1 - \exp\left(-A_o t^{1/2}\right) \right] + \left(K_s\right)_s t \tag{6}$$

where $A_o(t^{-\nu_2})$ depends upon hydraulic functions of the soil and it includes the truncation error, too, similarly to Swartzendrubers estimate of saturated conductivity $(K_s)_s t$. Substituting $4 K_s / 3 S_s$ for A_o we obtain a two-parameter equation. On the other hand, if we consider only the first four terms of a series expressing exp $(-A_o t^{\nu_2})_s$ we obtain an equation identical to Eq. (3).

Brutsaert's [1] 3-parameter equation is:

$$I = (K_{s})_{B} t + \frac{S_{B}^{2}}{B(K_{s})_{B}} \cdot \left\{ 1 - \frac{1}{\left[1 + (B(K_{s})_{B} t^{\nu_{2}})/S \right]^{2}} \right\}$$
(7)

He considered values of B=1/3, 2/3, or 1 each descriptive of the physical reality, but for most practical purposes he recommended B=1. Theoretical treatment has shown that the errors of estimation of S and K_s were the lowest of the treated Eq. [2].

All the equations mentioned were derived deterministically for trivial conditions $d\theta_i/dt=0$ and $d\pi_i/dz=0$, where π_i denotes physical charasteristics of soil as well as the functional relations of the retention curve or of the unsaturated conductivity. However, these conditions are not met in the field and further detailed study is needed.

The proper interpretation of the measured data requires the correction of the first reading of I_1 after the start of infiltration t_1 . The water applied on the surface as the reference at t = 0 is unknown due to the unknown surface storage and owing to our uncertainty in reading the reference surface pressure head at t = 0. This level was optimized through Eq. (1) applied to the first set of readings [2].

TESTING OF INFILTRATION EQUATIONS

Applicability of equations

The field data of I(t), after correction of the first reading were fitted to Eqs (1), (3), (6) and (7), and the appropriate infiltration parameters were evaluated. Already this first procedure brought some new views, mainly due to typical field conditions where the assumptions of the theoretical development were not met:

 a) Homogeneity of soil properties in the vertical does not exist and the deviations are mainly of stochastic character.

b) Initial soil water content is not constant with depth, partly due to a), partly due to a nonconstant boundary condition of both infiltration and evaporation, or evapotranspiration, respectively.

Philip's Eq. (1): In 30 % of instances the parameter A was negative. This is only acceptable if A is taken as a fitting parameter. The high percentage of non-applicability, not described in literature till now, is assumed to be the result of the very low structural stability even in the short time span of a single experiment. Roughly in 50 % of the same instances the values of parameter B of Brutsaert's Eq. were extremely low. Both the three-parameter Eqs (3) and (6) behaved for those instances, too, anomalously, having negative values of either the second or of the third parameter.

Brutsaert's Eq. (7): Parameter B takes over all anomalies of the field infiltration in about 16 % of instances, either $B \rightarrow 0$ or $B \approx 10^4$ to 10^{-6} . If B was very high, $B \approx 3$, then the parameters S_B and $(K_s)_B$ showed distinct deviation from estimates obtained by means of other equations.

Three-parametric Eq. (3): The parameter C_2

was negative in 41 % of instances and C_3 in 29 %. It means that for 29 % of the tests the estimation of K, was not possible. The combination of $C_2 < 0 \land C_3 < 0$ never occurred. In 3 % of instances the sorptivity C_1 was negative. Detailed analysis of the infiltration data has shown an error in those tests. In relation to K, by other equations, this procedure has given a good agreement, except of anomalies mentioned.

Swartzendrubers Eq. (6): In 29 % of tests, the equation was not applicable due to the negative value of parameter which leads to the estimate of K_{s} , analogically to Eq. (4).

Generally, the three-parametric Eqs (3) and (6) the best applicability have. They are simple to use in the fitting procedure. Next is the Brutsaert's equation, where the non-typical values of B exclude the equation from use in some instances. Here, and if the negative value C_3 occurs, we recommend to use the Philips Eq. (1) as a substitute, if applicable. Therefore, there is no recommendation for the use of only one equation. All equations used here should be tested, and the one well fitting to the measured data is finally applicable. On the other hand, if neither of Eqs (1), (3), (6) and (7) is applicable, we have either an error in the measuring procedure or there is such non-homogeneity in the soil profile, due to some local compaction, layering etc., that our concept of infiltration is invalidated. An increase in the number of parameters increases theoretically the accuracy of estimates of K_s , S, or, possibly also of K_i and of other physical characteristics, but, on the other hand, it contributes to the 'vulnerability' of the equation due to the heterogeneity of soil profile and the nonconstant character of the initial water content.

PDF of infiltration parameters

The infiltration parameters were first evaluated statistically. Since in about 30 % of the tests the parameters were physically nonrealistic or the equation was not applicable, we evaluated the full population of parameters and, in the alternative, the set was restricted and the tests with physically non-realistic parameters were deleted. In this shortened sample belong only those infiltration tests

where all the equations are applicable. It means that we are comparing the statistical characteristics all the time on the same population and once for a full set with 70 infiltrations, and once for a shortened set with 32 infiltrations only. For both sets, the parameters of sorptivity S (cm min^{-1/2}), and saturated hydraulic conductivity K. (cm min⁻¹) are in the top part of Table 1. With the exception of K, for the full set according to Philip's Eq. (1), the mean values of parameters gained by the equations tested are comparable and there was no tendency to overor underestimation of parameters according to individual equations. The dispersion of K_s was much greater than that of S in all the instances. In the bottom part of Table 1 we find the results of the testing of PDF. For Philip's K and Brutsaert's S and K_s , the full set was not evaluated due to the strong deviation of data from the physical reality. Let us concentrate therefore upon the shortened set. Two types of PDF are

included in the table, the first one for the Kolgomorov-Smirnov test, the second one for the χ^2 test.

The comparison of PDF for the individual parameters and equations shows that the real PDF of a certain physical parameter is deformed by the summation error $\varepsilon_{\Sigma} = \varepsilon_1 + \varepsilon_2$, where ε_1 for the error due to the approximative character of the equation. Error ε_2 exists owing to the declination of the field reality from the assumed soil physical homogeneities in the theoretical development of equations. For the shortened set, the most frequent is the log-normal distribution. The same was found for Nigerian soils [3]. We are allowed to conclude that the log-normal distribution is a well acceptable approximation for PDF of S and K_{s} . However, each equation offers a slightly different PDF due to the above mentioned errors. It follows, that the published studies on PDF of hydraulic functions can not be understood as the determination of real PDF of a particular hydraulic property of soils.

Parameter		Philip Eq. (1)		3-para Eq. (3)		Swartzendruber Eq. (6)	Brutsaert Eq. (7)	
		S	$K_s = 1.5A$	S	K _s	Ks	S	K _s
\overline{x}	FULL	0.456	0.012	0.486	0.042	0.029	0.536	0.035
\overline{x}	SHORT	0.412	0.040	0.459	0.053	0.041	0.489	0.040
SD	FULL	0.280	0.033	0.327	0.030	0.029	0.363	0.025
SD	SHORT	0.259	0.037	0.282	0.031	0.026	0.300	0.029
q ₃	FULL	0.721	-0.350	1.157	1.128	0.916	1.49	1.456
q ₃	SHORT	0.813	1.661	0.812	0.874	1.209	0.821	0.977
q ₄	FULL	-0.034	4.432	1.386	1.008	1.241	3.534	2.239
q ₄	SHORT	-0.232	2.284	-0.47	-0.004	0.987	-0.395	0.263
DISTRIBUTION		w	-	Е	В	w	-	-
FULL		G	-	G	W	E	-	-
SHORT		LG	LG	LG	w	В	LG	w
		E	G	G	N	w	G	В
FULL SHORT	- full pupulation - shortened population without the tests with physically not real parameters in some of the						ne	

SHOKI	equations	ation without the tests with p	nysically not real parameters i	ii some of
x	- mean			
SD	- standard deviati	on		
q ₃	- skewness (3rd n	noment)		
94	- kurtosis (4th mo	oment)		
Distribution	N - Normal,	LG - Log-normal,	W - Weibull's,	
	G - Gama,	B - Beta,	E - Erlang's	

The determined PDF is only an approximation which is more distant from the reality the greater is the summation error ε_{Σ} . Geostatistical evaluation of *S* and *K_s* by variograms shows that for 7.5 m no spacial influence is detectable.

Statistical evaluation of saturated conductivity estimates

For the sake of completeness, the estimates of K_s as obtained by a) infiltration with Brutsaert's Eq., b) core samples in laboratory, c) Guelph permeameter, were statistically evaluated.

The Guelph permeameter gives in average the smallest values of hydraulic conductivity, while the surface infiltrometer produces the highest mean. The t - test can be applied to the logarithms of conductivities, to prove the relevance of the differences in the mean (Table 2). The differences between the methods are highly significant. Empirical variograms of the hydraulic conductivites were calculated using

Table 2. Basic statistics for K3, plot A. Units: m, s

standard software. They show only weak autocorrelation between the nearest neighbours in the grid i.e. to the distance of 7.5 m. Spherical model was arbitrary chosen to fit the empirical variograms.

With the help of these variograms, block kriging was carried out and the contour maps obtained. The resulting pattern of areal the variability of K_s is rather erratic. While the methods a) and b) seem to give somewhat similar patterns, the Guelph permeameter method indicates spots of higher or lower permeability elsewhere.

Linear regressions were calculated for corresponding couples of $\ln K$ values at the same points, to demonstrate the degree of mutual correlation between the methods. The resulting correlation coefficients were very low, and with respect to the method c) even negative, which quantifies the impression from the contour maps (see Tables 1 and 2).

The differences between the three methods could be partially explained by the following

Property and method		No. of observations	Mean	Stdandard deviation	Skewness
К	(a)	70	5.84 10-6	4.13·10 ⁻⁶	1.39
	(b)	51	3.43.10-6	2.63.10-6	1.32
	(c)	54	2.68.10-6	3.32.10-6	2.57
1n 10 ⁶ K	(a)	70	1.509	0.780	-0.74
	(b)	51	0.958	0.764	-0.01
	(c)	54	0.457	1.009	0.34
Φm	(c)	54	3.04 10-7	3.76.10-7	2.58
S	(c)	54	3.46 10-4	1.68.10-4	0.93
θ_f (% vol.)	(b)	54	35.1	2.5	0.90
θ_i (% vol.)	(b)	54	17.7	2.9	0.55

Table 3. Results of t-test, 1n Ks - plot A

Method	Degree of freedom	Difference of means	Criterion Diff/StdErr	t-quantil (d.f., level)
(a) vs. (b)	119	0.552	3.874	2.6181 (118.0.01)
(a) vs. (c)	122	1.052	6.551	2.6167 (122.0.01)
(b) vs. (c)	103	0.501	2.854	2.6249 (102.0.01)

considerations (Table 3):

The higher value of K_s obtained from ponded infiltration could be explained by the presence of vertical macropores (they are more numerous just at the soil surface). On the contrary, the Guelph permeameter measurements are based mainly upon the horizontal infiltration into the walls of the borehole, in our experiments at the depths of 10-20 cm below the surface, where the macropores (preferably in horizontal direction) are less frequent, and where the soil could be also compacted by agricultural machinery. The results of laboratory measurements (intermediate values) could be considered as very successful with regard to the well known problems with interpretation of measured values.

Using the Guelph permeameter, the possibility of smearing the boreholes by the auger must also be taken into consideration, however the walls of the holes were roughened before the measurement with a brush each time.

Time variability of infiltration characteristics

As it was concluded in the previous chapter, a stochastic variation of infiltration characteristics has been obtained. One of the problems seems to be the structural instability of the surface part of A horizon. Then time variation of the infiltration and the physical characteristics is very important feature. Repeated series of 20 infiltration tests on plot B have been performed.

The sets of data were analysed using the same procedures as in the previous chapter. The equations of Philip Eq. (1), three-parametric Eq. (3) and of Swartzendruber Eq. (6) were used. The results are given in Table 4. In addition to those data, infiltration rate after 30 min was statistically analysed too, see Table 5.

If we compare the results according to quasi steady infiltration rate and saturated hydraulic conductivity, the mean values have the decreasing tendency with time. The strongest decreasing is between the first and the second set of tests. The values of sorptivity depend on the initial moisture content, that's why it is impossible to evaluate the time dependence of these variables correctly.

The strong drop of both the saturated conductivity and the infiltration rate after 30 min between the first and the second series documents the fact of fast decay of big pores and macropores which appeared as the consequence of ploughing and soil cultivation in the spring. The decay of K_s occurred mainly within the first vegetation season. The next spring infiltration measurements do not confirm the generally accepted theory that porosity and hydraulic conductivity are positively influenced by winter meteorological conditions. Our observations show that the decrease in conductivity continues even in

Para-		S _{ph} /series			S _{3p} /series			S _{sw} /series		
meter	1	2	3	1	2	3	1	2	3	
x	3.53	0.91	0.60	2.46	0.99	0.66	2.46	0.99	0.66	
X _{max} .	12.93	3.35	1.63	6.80	3.29	2.08	6.80	3.29	2.08	
X _{min.}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
SD _x	3.20	0.88	1.94	1.94	0.93	0.51	1.94	0.93	0.51	
	A _{ph} /series			K _{3p} /series		K _{sw} /series				
\overline{x}	0.97	0.26	0.09	1.08	0.35	0.15	1.03	0.37	0.12	
X.max.	3.37	1.07	0.30	3.41	1.43	0.40	3.41	1.24	0.33	
Xmin	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
SD,	P-78	0.29	0.08	0.84	0.34	0.11	0.84	0.31	0.09	

Table 4. List of parameters of infiltration equations

 S_{ph} - sorptivity from Philip's Eq. (1), S_{3p} - sorptivity from 3 - parameter Eq. (3), S_{sw} - sorptivity from Swartzendruber's Eq. (3), similar for saturated conductivity K, or for A parameter, Eq. (1), respectively, \overline{x} - average, S_{Dx} - standard deviation, K_{sw} - hydraulic conductivity from 3-parametric Philip equation, K_{sw} - hydraulic conductivity from Swartzendruber equation.

Parameter		v (cm min. ⁻¹)	
i alameter	1 series	2 series	3 series
x	1.25	0.39	0.16
Xmax	3.50	1.30	0.41
X _{min}	0.26	0.06	0.035
R	3.25	1.24	0.375
σ	0.73	0.22	0.09
s ²	0.80	0.09	0.01
SD,	0.894	0.299	0.103
C,	0.715	0.773	0.629
с,	1.201	1.627	0.889

Table 5. Infiltration rate after 30 min. Statistical characteristics

 \overline{x} - average, R - range, σ - average deviation, s² - dispersion, SD_x - standard deviation, c_v - variance, c_s - asymmetry.

winter time. On the other hand, the next spring data are still more than 3 times higher than the ones obtained after 4 years of grass cultivation and harvesting.

CONCLUSIONS

The comparative study on the applicability of infiltration equations has shown that the 3-parameters Eqs (3) and (6) were best applicable for infiltration, tests. Among the PDF fitting to hydraulic characteristics, the log-normal distribution is a well acceptable approximation. For the geometry of infiltration we found in the weakly aggregated soils a feable spatial covariance for the distance of 7.5 m. Long time variability of hydraulic characteristics is closely related to the low stability of soil structure. There was a steep decrease of hydraulic characteristics when two sets of infiltration tests were compared just after ploughing and the preparation of seedbed in spring, and the other one on the same locations, but with grass cover in autumn.

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